AC Loss in Fine Superconducting Filaments under Reversible Motion of Flux Lines

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Abstract— **The magnetic flux distribution and the energy loss density under an external AC magnetic field were theoretically analyzed for a fine superconducting filament. The Campbell model for the force-displacement relation and the continuity equation for flux lines were solved numerically. It was shown that the AC loss takes a much smaller value due to the reversible motion of flux lines than predicted by the critical state model.**

Keywords— **reversible flux motion, reduction of hysteresis loss, Bi-2223 silver sheathed tape**

I. Introduction

FURTHER reduction in the AC loss is a key issue for applying oxide superconductors to practical AC applications. It has been shown that the energy loss density can be reduced from the prediction of the critical state model due to the reversible flux motion at 77.3 K even in an oxide superconductor when the filament thickness is reduced to the order of micrometer [1]–[3]. There is no rigorous theoretical work to be compared with the experimental results, since the existing theoretical treatment has been done only the case where a small AC magnetic field is superposed on a large DC magnetic field [4].

In this work, the reversible flux motion is theoretically analyzed for the case where only an AC magnetic field is applied on a thin superconductor. The magnetic field dependence of the critical current density is also taken into account. The theoretical result is compared with the experimental result on a Bi-2223 multifilamentary tape with fine filaments [2].

II. Theory

For simplicity, it is assumed that the AC magnetic field $B_\mathrm{m}\cos\omega t$ is applied along the z-axis parallel to an infinite superconducting slab occupying $0 \le x \le d$. From symmetry we have to consider only one half, $0 \le x \le d/2$. It is assumed, for simplicity, that the flux distribution in initial state, B_ini ($\omega t = -\pi$) is in the critical state as shown in Fig. 1. When the AC magnetic field is increased, the flux distribution varies and the corresponding displacement of flux lines, $u(x)$, can be obtained from the continuity equation for flux lines:

$$\frac{\partial}{\partial x}(Bu) = -(B - B_\mathrm{ini}). \tag{1}$$

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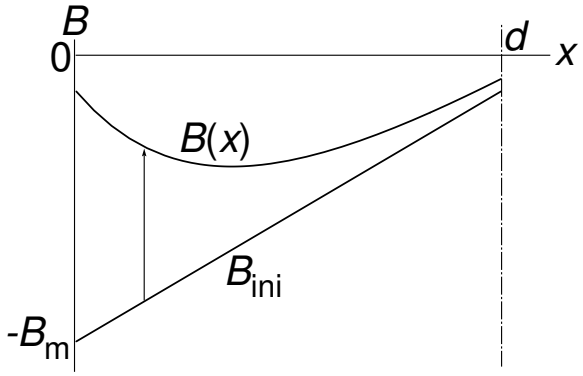

Fig. 1. Variation in the flux distribution in a superconducting slab from the initial state.

The current density, J, is related to the magnetic flux density by

$$\frac{1}{\mu_0}\frac{\partial B}{\partial x} = J. \tag{2}$$

J is equal to J_c in the initial state, and changes with the displacement of flux lines as [5]

$$J = -J_\mathrm{c}\left[1 - 2\exp\left(\frac{-u}{2d_\mathrm{i}}\right)\right], \tag{3}$$

where d_i is the interaction distance representing the half size of the pinning potential. At high fields, d_i is given as a_f/ζ, where a_f is the flux line spacing and ζ is a constant ranged from 4 to 2π depending on the kind of pinning centers. In the low field limit the above expression of d_i diverges. In order to avoid such an unphysical situation, it is assumed that the upper limit of d_i is given by 1/10 of specimen size d. For the magnetic field dependence of the critical current density, J_c, modified Kim's model [2] is assumed:

$$J_\mathrm{c}(B) = \frac{\alpha_0}{|B| + \beta}\left(1 - \frac{|B|}{B_\mathrm{i}}\right), \tag{4}$$

where α_0, β are parameters and B_i is the irreversibility field at which J_c reaches zero. The boundary conditions are $B(0) = B_\mathrm{m}\cos\omega t$ at the surface ($x = 0$) and $\mathrm{d}B/\mathrm{d}x = \mu_0 J_\mathrm{c}$ at the center of the slab ($x = d$). The magnetization, M, is calculated from the spatial average of the magnetic flux density over the slab for a half cycle from the initial state during which the surface field varies from $-B_\mathrm{m}$ to B_m. For simplicity, the minor curve of M in the next half cycle is assumed to be symmetry to that in the previous half cycle so that the minor magnetization curve closes. Then, the energy loss density is approximately calculated from the area of the closed magnetization curve.

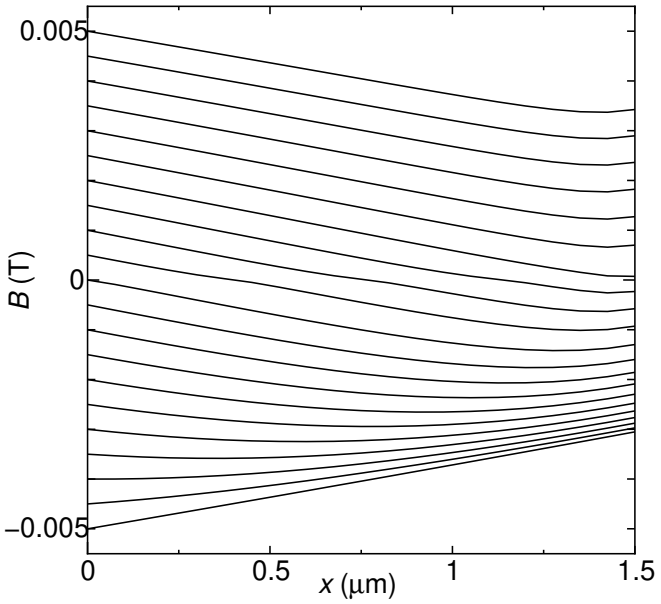

Fig. 2. Calculation result of the magnetic flux distribution in the superconducting slab.

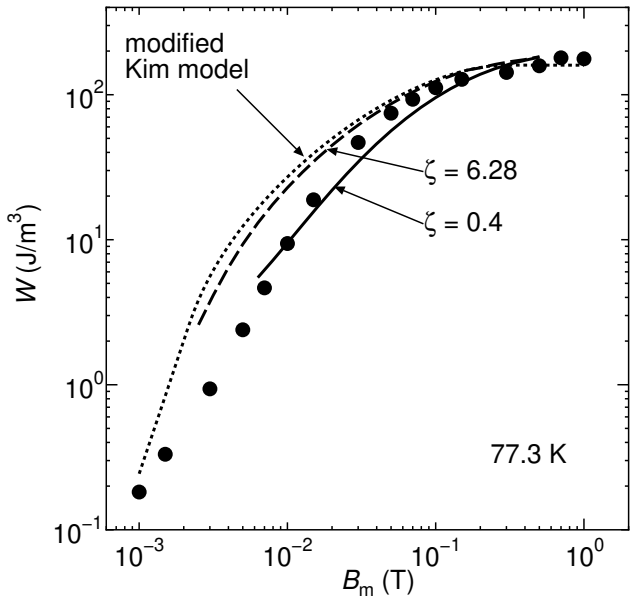

Fig. 3. Comparison of several calculated results of the energy loss density. Closed symbols are experimental results [2].

TABLE I
THE PARAMETERS USED IN THE THEORETICAL CALCULATION.

α_0	β	B_i	d	ζ
3×10^7	2.5×10^{-2}	0.3	3.0×10^{-6}	0.4–6

III. Result and Discussion

The calculated result of the variation in the magnetic flux distribution in the superconducting slab for $B_\mathrm{m} = 5$ mT is shown in Fig. 2. The parameters used in the calculation are listed in Table I, which are determined so as to obtain a good agreement for the major magnetization curve between experiment on a Bi-2223 multifilamentary tape at 77.3 K and theory [2]. The effect of reversible flux motion is significant when the variation in the external magnetic field from the initial state is small. On the other hand, the flux motion is almost irreversible with a change in J from J_c to $-J_\mathrm{c}$ in the almost whole region of the filament when the variation in the external magnetic field is large.

In Fig. 3, the calculated results of the energy loss density, W, are shown as a function of the AC magnetic field amplitude, B_m, and compared with the measured result of Bi-2223 multifilamentary tape at 77.3 K. In the figure, the solid, chained and doted lines represent the results for $\zeta = 0.4$, $\zeta = 6.28$ and the prediction of the irreversible modified Kim's model, respectively. These three calculated results agree with the measured result represented by symbols at large B_m, where the flux motion is almost irreversible. It is to be noted that W is saturated at large B_m, since B_m becomes larger than B_i.

On the other hand, the reversible flux motion becomes significant when ζ is small, resulting in smaller W than the prediction of completely irreversible modified Kim's model at medium AC magnetic field amplitudes. However, $\zeta = 0.4$ obtained so as to fit with the measurement is too small in comparison with the theoretically predicted value. The reason is still open.

The calculation has not yet completed for smaller AC magnetic field amplitudes, since the assumption that the initial magnetic flux distribution is in the critical state is not fulfilled. Therefore, it is necessary to calculate the steady-state distribution after sufficient cycles started from the virgin state.

References

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