MAGNETIC STRUCTURE OF FLUX LINE LATTICE

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ABSTRACT

Magnetic structure of a perfect triangular lattice at relatively low fields is numerically calculated by superposing isolated flux lines based on the Ginzburg-Landau theory. It is found that the magnetic flux density is fairly uniform spatially and the difference between the maximum and minimum values is 25 % when the fluxoid spacing, a_{f}, is equal to the penetration depth, λ, and the G-L parameter $\kappa = 50$. The obtained magnetic structure is expressed in the Fourier series and the expansion summed terms are obtained as a function of κ and a_{f}/λ. It is seen that the number of coefficients needed for reconstruction becomes small with increasing magnetic flux density.

1. Introduction

The exotic nature of flux lines in two-dimensional oxide superconductors has been extensively argued based on various experiments. For example, Cubitt *et al.*[1] found for a Bi-2212 single crystal that the neutron diffraction pattern due to the flux lines along the c-axis disappeared at around 60 mT even at low temperatures. The observation by the Lorentz microscope[2] clarified that the image of flux lines still exists even above the irreversibility line in the high temperature region. However, these kinds of observation are directly influenced by the magnetic structure of flux lines. For example, it is argued that a severe overlap of magnetic field due to very long penetration depth limits the resolution of observation. Therefore, it is necessary to understand the detailed magnetic structure of flux line lattice. For this purpose we calculate theoretically the magnetic structure of a perfect flux line lattice in an isotropic superconductor. This result is also applicable to the case where the flux lines are directed to the c-axis in a tetragonal oxide superconductor.

2. Theory

The magnetic structure of an isolated flux line is calculated by solving the Ginzburg-Landau equation numerically. If r is the radius from the center of the flux line, all the physical quantities depend only on r. Using Abrikosov's units, the G-L equations are reduced as[3]

$$-\frac{1}{\kappa^2\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\rho\frac{\mathrm{d}f}{\mathrm{d}\rho}\right)+Q^2 f = f - f^3, \tag{1}$$

$$\frac{\mathrm{d}}{\mathrm{d}\rho}\left[\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}(\rho Q)\right] = Qf^2, \tag{2}$$

where $\rho = r/\lambda$, f is the normalized order parameter and $Q = |\boldsymbol{a} - \nabla\phi/\kappa|$ with $\boldsymbol{a}$ and ϕ denoting the normalized vector potential and the phase of the order parameter, respectively. The normalized magnetic flux density is given by

$$b = -\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}(\rho Q)\,. \tag{3}$$

Equations (1) and (2) are numerically solved under the boundary conditions of $f = 0$ at $\rho = 0$ and $f \to 1$ and $Q \to 0$ at $\rho \to \infty$.

The most stable structure of flux line lattice is the triangular lattice. This can be approximately calculated by superposing isolated flux lines. To fulfill the condition of superposition, it is necessary that the normal cores of flux lines do not overlap appreciably. This condition is expressed that the distance between the nearest flux lines is longer than 4ξ with ξ denoting the coherence length,[4] i.e., $B < \phi_0/8\sqrt{3}\xi^2 = (\pi/4\sqrt{3})B_{c2}$. The number of superposed flux lines is at least 100×100 in the X-Y plane and the magnetic structure of the unit cell in the central part is calculated. Therefore, the magnetic flux density of the flux line at the position of $r = 50a_\mathrm{f}$ from its center should be sufficiently small. This condition is expressed as $50a_\mathrm{f} \gg \lambda$.

The magnetic structure of the triangular flux line lattice is expressed in the Fourier series as

$$\begin{aligned} b(x,y) = B\{1 \;&+\; 2F_{10}[\cos y + \cos(x - y/2) + \cos(x + y/2)] \\ &+\; 2F_{11}[\cos(x + 3y/2) + \cos 2x + \cos(-x + 3y/2)] \\ &+\; 2F_{20}[\cos 2y + \cos(2x - y) + \cos(2x + y)] \\ &+\; 2F_{21}[\cos(2x + 5y/2) + \cos 3(x + y/2) + \cos(3x - y/2) + \\ &\qquad \cos(-2x + y/2) + \cos(-x + 5y/2) + \cos(x + 5y/2)] \\ &+\; 2F_{30}[\cos 3y + \cos 3(x - y/2) + \cos 3(x + y/2)] \\ &+\; 2F_{31}[\cos(-2x + 3y) + \cos(4x - 3y/2) + \cos(2x + 7y/2)] \\ &+\; \}\cdots \end{aligned} \tag{4}$$

where $x = 2\pi X/a_\mathrm{f}$ and $y = 4\pi Y/\sqrt{3}a_\mathrm{f}$ and F_{10}, $F_{11}\cdots$ are Fourier coefficients. The Fourier coefficients can be calculated using the obtained magnetic structure.

3. Results and Discussion

The magnetic structure inside the unit cell of flux line lattice is shown by dotted lines in Fig. 1 for (a) $a_\mathrm{f}/\lambda = 1$ and (b) $a_\mathrm{f}/\lambda = 1/3$ and $\kappa = 50$. It is found that the magnetic flux density is fairly uniform spatially and the difference between maximum and minimum values, δb, is 25 % even for $a_\mathrm{f}/\lambda = 1$. Cubitt *et al.* insisted that the transition between two- and three- dimensional flux lines occured at 60 mT at low

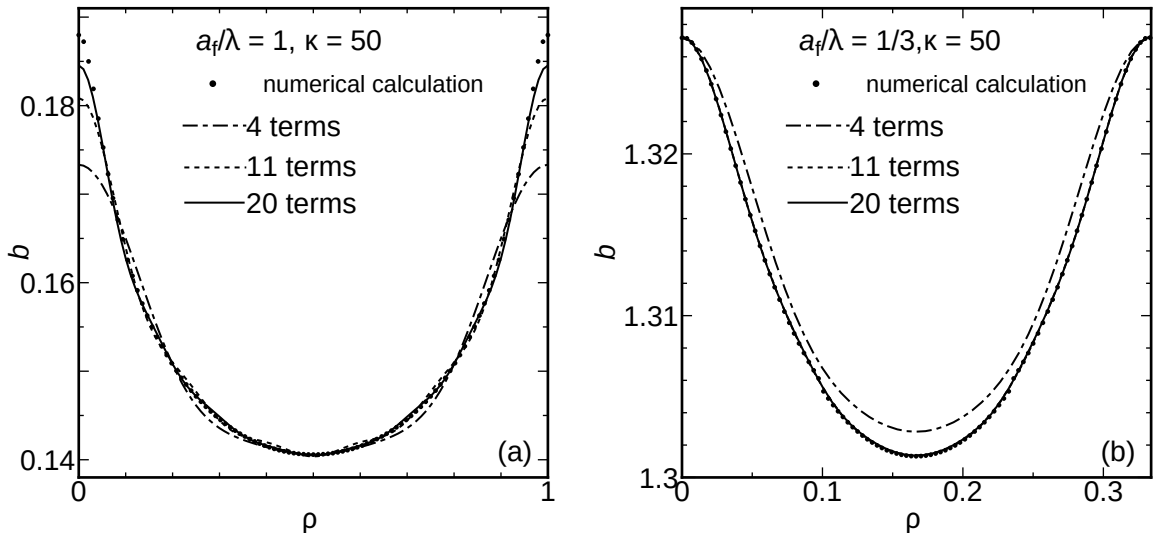

Figure 1: Numerically calculated magnetic structure along the close-packed direction inside the unit cell of flux line lattice and result of summed finite terms of Fourier series for $\kappa = 50$ at (a) $a_{\rm f}/\lambda = 1$ and (b) $a_{\rm f}/\lambda = 1/3$.

temperatures in Bi-2212 superconductor. However, the ratio of $a_{\rm f}/\lambda$ at 60 mT is 0.77 and δb is only 16 % of the maximum magnetic flux density, $b_{\rm m}$, where $\lambda(0) = 260$ nm is assumed inside the a-b plane. If the temperature dependence of λ is assumed as $\lambda(T) = \lambda(0)\left[1-(T/T_{\rm c})^4\right]^{-1/2}$, the magnetic field at which δb is 16 % of $b_{\rm m}$ is shown in Fig. 2. This result at low temperatures is similar to the magnetic field where the neutron diffraction pattern disappears. Therefore, the disappearance of the neutron diffraction pattern might be caused by the small spatial difference due to a significant overlap of flux lines and this seems to be a candidate for the mechanism of the disappearance of pattern.

The results of the magnetic flux density calculated using several terms of Fourier series are also shown in Fig. 1 (a) and (b). Examples of Fourier coefficients are given in Table 1. It is found that the deviation of the expression by the finite terms of the Fourier series from the numerical calculation becomes small according to increasing magnetic flux density. That is, the deviation remains 1 % at low magnetic field in Fig. 1(a) even when 20 terms are summed up, while the deviation is small enough for the summation of 11 terms at high magnetic field in Fig. 1(b). On the other hand, it is necessary to increase the number of summed terms for higher κ values.

4. Conclusion

In this paper, the magnetic structure of a perfect triangular lattice is numerically calculated by superposing isolated flux lines based on the Ginzburg-Landau theory. The obtained magnetic structure is expressed in the Fourier series. The following results are obtained. (1) The magnetic flux density is fairly uniform in space even at

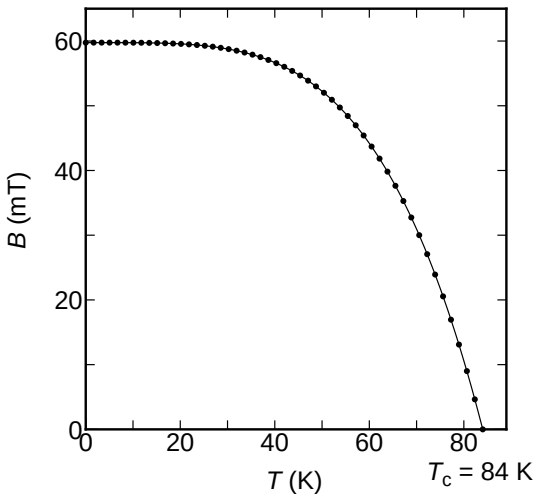

Figure 2: The temperature dependence of the magnetic field where δb is 16 % of $b_{\rm m}$.

Table 1: Fourier coefficients

	$\kappa = 50$		$\kappa = 100$
F_{hk}	$a_{\rm f}/\lambda = 1$	$a_{\rm f}/\lambda = 1/3$	
F_{10}	1.83×10^{-2}	1.76×10^{-3}	1.97×10^{-3}
F_{11}	5.94×10^{-3}	3.97×10^{-4}	5.81×10^{-4}
F_{20}	4.38×10^{-3}	2.44×10^{-4}	4.11×10^{-4}
F_{21}	2.36×10^{-3}	8.38×10^{-5}	2.01×10^{-4}
F_{30}	1.76×10^{-3}	5.25×10^{-5}	1.43×10^{-4}
F_{31}	1.24×10^{-3}	3.37×10^{-5}	9.60×10^{-5}
F_{32}	1.12×10^{-3}	2.94×10^{-5}	8.57×10^{-5}
F_{40}	8.55×10^{-4}	1.95×10^{-5}	6.36×10^{-5}
F_{41}	6.74×10^{-4}	1.15×10^{-5}	4.94×10^{-5}
F_{42}	5.84×10^{-4}	6.90×10^{-6}	4.25×10^{-5}
F_{50}	4.48×10^{-4}	1.04×10^{-6}	3.23×10^{-5}
F_{51}	3.97×10^{-4}	-8.69×10^{-7}	2.83×10^{-5}
F_{52}	3.74×10^{-4}	-1.39×10^{-6}	2.66×10^{-5}
F_{53}	3.16×10^{-4}	-1.38×10^{-6}	2.22×10^{-5}
F_{60}	2.44×10^{-4}	5.55×10^{-7}	1.64×10^{-5}
F_{61}	2.32×10^{-4}	7.47×10^{-7}	1.53×10^{-5}
F_{62}	2.11×10^{-4}	1.26×10^{-6}	1.36×10^{-5}
F_{63}	3.16×10^{-4}	-1.37×10^{-6}	2.22×10^{-5}
F_{64}	1.42×10^{-4}	2.83×10^{-6}	7.54×10^{-6}
F_{70}	1.37×10^{-4}	2.28×10^{-6}	7.18×10^{-6}

low magnetic fields in Bi-2212 superconductor. (2) The deviation of the approximation using finite terms of the Fourier series from the numerical calculation becomes small according to increasing magnetic field. The number of summed terms needed for a sufficient agreement becomes larger for higher κ values.

5. References

1. R. Cubitt, E. M. Forgan, G. Yang, S. L. Lee, D. Mck. Paul, H. A. Mook, M. Yethiraj, P. H. Kess, T. W. Li, A. A. Monevsky, Z. Tarnawski and K. Mortensen, *Nature* **365** (1993) 407.
2. K. Harada, T. Matsuda, H. Kasai, J. B. Bonevich, T. Yoshida, U. Kawabe and A. Tonomura, *Phys. Rev. Lett.* 71 (1993) 3371.
3. A.A. Abrikosov : Soviet Phys. JETP **15** (1957) 1175.
4. M. Nozue, K. Noda and T. Matsushita, to be published in *Adv. in Superconductivity VIII* (Springer, Tokyo, 1996)