# UPPER CRITICAL FIELD OF MULTILAYERS WITH DIFFERENT COHERENCE LENGTHS

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## ABSTRACT

The upper critical field of multilayers with different coherence lengths under a normal magnetic field is theoretically investigated using the Ginzburg-Landau theory with a simple assumption on the spatial variation in the order parameter. The case of Nb-Ti with artificially introduced Nb as pinning centers is treated as an example. It is found that the upper critical field is seriously degraded by the proximity effect. The present result agrees well with the result of microscopic theory of Takahashi and Tachiki. The method of the Ginzburg-Landau theory with an approximate expression of the order parameter seems to be useful for a theoretical estimation on the pinning property which is difficult to treat with the microscopic theory.

**KEYWORDS:** upper critical field, multilayer, Ginzburg-Landau theory, Nb-Ti, Nb artificial pinning center

## INTRODUCTION

Improvement of the critical current density, $J_c$, by introduction of artificial pinning centers was succeeded for the case of Nb-Ti with artificial Nb pinning centers [1]. Although very high $J_c$ value was achieved up to a medium magnetic field, the upper critical field was degraded from the starting Nb-Ti. This degradation was interpreted to be caused by the proximity effect [2]. That is, it is considered that the induced superconductivity brings about an increase in the kinetic energy proportional to $\xi^2$ through a steep spatial variation in the order parameter. As a result, the superconductivity in the region which is in contact with a material with the long coherence length is degraded to avoid the energy increase and the upper critical field is reduced.

The improvement of the pinning property and the reduction in the upper critical field are trading off and it is very important to find the optimum condition for the application. For this purpose it is necessary to understand the superconducting property of the superconductor with a pinning material. In this paper the upper critical field is investigated using the Ginzburg-Landau theory with an approximate formula of the order parameter for multilayers composed of Nb-Ti and Nb. The obtained result is compared with the microscopic theory of Takahashi and Tachiki [3].

## THEORY

The superconducting property under a magnetic field is described correctly by the Ginzburg-Landau theory. According to this theory, the free energy density is given by

$$F_s(B) = F_n(0) + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{B^2}{2\mu_0} + \frac{\hbar^2}{2m^*}(\nabla|\Psi|)^2 + \frac{\mu_0}{2}\lambda^2\left(\frac{|\Psi_\infty|}{|\Psi|}\right)^2 \boldsymbol{J}^2. \quad (1)$$

In the above $F_n(0)$ is the free energy density in the normal state in the absence of magnetic field, $\Psi$ is the order parameter, $\Psi_\infty$ is its equilibrium value, $m^*$ is the mass of superconducting electron, $\lambda$ is the penetration depth, $\boldsymbol{J}$ is the current density and $\alpha$ and $\beta$ are coefficients. Here we assume for simplicity an ideal multilayered structure as shown in Fig. 1. The thicknesses of Nb and Nb-Ti layers are denoted by $d_N$ and $d_{NT}$, respectively. It is assumed that the magnetic field is applied normal to the layer structure, i.e., parallel to the $z$-axis. It is expected that the order parameter in

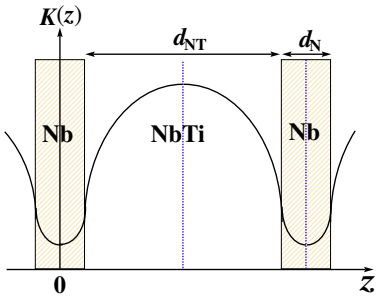


Fig. 1. Variation of function $K(z)$ in multilayer of Nb-Ti and Nb.

in the vicinity of the upper critical field is approximately described in the separated form of

$$\frac{|\Psi|^2}{|\Psi_\infty|^2} = K(z)\left[1-\frac{1}{3}\left\{\cos\frac{2\pi}{a_{\rm f}}\left(\frac{x}{\sqrt{3}}+y\right) + \cos\frac{2\pi}{a_{\rm f}}\left(\frac{x}{\sqrt{3}}-y\right)+\cos\frac{2\pi}{a_{\rm f}}\left(\frac{2}{\sqrt{3}}x\right)\right\}\right], \tag{2}$$

where $a_{\rm f}=(2\phi_0/\sqrt{3}B)^{1/2}$ is the flux line spacing with $\phi_0$ denoting the flux quantum and $K(z)$ shows a modification along the $z$-axis. We assume $K(z)$ as

$$K(z) = a\left(1-b\cos\frac{z}{\xi_{\rm N}}\right); \qquad 0\le z\le\frac{d_{\rm N}}{2}, \tag{3a}$$

$$= a'+b'\cos\frac{1}{\xi_{\rm NT}}\left(z-\frac{d_{\rm N}+d_{\rm NT}}{2}\right); \qquad \frac{d_{\rm N}}{2}\le z\le\frac{d_{\rm N}+d_{\rm NT}}{2}, \tag{3b}$$

where $a$, $b$, $a'$ and $b'$ are constants. $\xi_{\rm N}$ and $\xi_{\rm NT}$ are the coherence lengths of Nb and Nb-Ti, respectively. It is considered that the expression of Eq. (3) for $K(z)$ is a good approximation when $d_{\rm N}$ and $d_{\rm NT}$ are comparable to or smaller than $\xi_{\rm N}$ and $\xi_{\rm NT}$, respectively.

It is assumed here that the thermodynamic critical field, $B_{\rm c}$, and the coupling constant of the Cooper pair are the same between Nb and Nb-Ti, since the critical temperature is the same. Then, the boundary conditions of de Gennes [4] are written that $|\Psi|^2$ and $\xi^2(\partial|\Psi|^2/\partial z)$ are continuous at the boundary of the two layers, $z=d_{\rm N}/2$. From these conditions $a'$ and $b'$ are expressed using $a$ and $b$ as

$$a' = a\left\{1-b\left(\cos\frac{d_{\rm N}}{2\xi_{\rm N}}+\frac{\xi_{\rm N}}{\xi_{\rm NT}}\cdot\sin\frac{d_{\rm N}}{2\xi_{\rm N}}\cot\frac{d_{\rm NT}}{2\xi_{\rm NT}}\right)\right\}, \tag{4a}$$

$$b' = a\,b\,\frac{\xi_{\rm N}}{\xi_{\rm NT}}\cdot\frac{\sin(d_{\rm N}/2\xi_{\rm N})}{\sin(d_{\rm NT}/2\xi_{\rm NT})}. \tag{4b}$$

If we confine ourselves to the case where the layer thicknesses are sufficiently shorter than $\lambda$, the magnetic structure is approximated to be unchanged along the $z$-axis. Then, the energy of magnetic field given by the fourth term in Eq. (1) can be omitted, since this is a constant. In addition, this leads to that the current flows only in the $x$-$y$ plane. Since our interest is confined to the vicinity of the upper critical field, the small term of the order of $|\Psi|^4$ can be neglected. Then, the free energy density is written in the form of a product of $a$ and a function only of $b$ as

$$\begin{aligned}\langle F_{\rm s}(B)-F_{\rm n}(B)\rangle &\simeq \frac{B_{\rm c}^2 a}{\mu_0}\left[-\frac{3}{4}+\frac{3bd_{\rm NT}}{4(d_{\rm N}+d_{\rm NT})}\cos\frac{d_{\rm N}}{2\xi_{\rm N}}+\frac{3b\xi_{\rm N}d_{\rm NT}}{4\xi_{\rm NT}(d_{\rm N}+d_{\rm NT})}\sin\frac{d_{\rm N}}{2\xi_{\rm N}}\cot\frac{d_{\rm NT}}{2\xi_{\rm NT}}\right.\\
&+\frac{7.26}{a_{\rm f}^2}\left(\frac{\xi_{\rm N}^2 d_{\rm N}+\xi_{\rm NT}^2 d_{\rm NT}}{d_{\rm N}+d_{\rm NT}}+\frac{2b\xi_{\rm N}(\xi_{\rm NT}^2-\xi_{\rm N}^2)}{d_{\rm N}+d_{\rm NT}}\sin\frac{d_{\rm N}}{2\xi_{\rm N}}-\frac{b\xi_{\rm NT}^2 d_{\rm NT}}{d_{\rm N}+d_{\rm NT}}\cos\frac{d_{\rm N}}{2\xi_{\rm N}}\right.\\
&-\left.\frac{b\xi_{\rm N}\xi_{\rm NT}d_{\rm NT}}{d_{\rm N}+d_{\rm NT}}\sin\frac{d_{\rm N}}{2\xi_{\rm N}}\cot\frac{d_{\rm NT}}{2\xi_{\rm NT}}\right)+\frac{1}{(d_{\rm N}+d_{\rm NT})}\left\{\xi_{\rm NT}\sqrt{c^2-R^2}\tanh^{-1}\left(\frac{c-R}{\sqrt{c^2-R^2}}\tan\frac{d_{\rm NT}}{4\xi_{\rm NT}}\right)\right.\\
&-\left.\left.\xi_{\rm N}\sqrt{1-b^2}\tan^{-1}\left(\frac{1+b}{\sqrt{1-b^2}}\tan\frac{d_{\rm N}}{4\xi_{\rm N}}\right)\right\}\right],\end{aligned} \tag{5a}$$

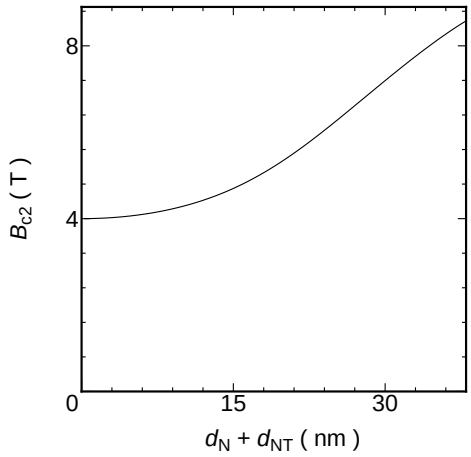


Fig. 2. Dependence of upper critical field on thickness of layers at $T = 4.2$ K for $d_{\rm N}/d_{\rm NT} = 0.25$.

where $R$ and $c$ are defined by

$$R = 1 - b\left(\cos\frac{d_{\rm N}}{2\xi_{\rm N}} + \frac{\xi_{\rm N}}{\xi_{\rm NT}}\sin\frac{d_{\rm N}}{2\xi_{\rm N}}\cot\frac{d_{\rm NT}}{2\xi_{\rm NT}}\right), \tag{5b}$$

$$c = b\,\frac{\xi_{\rm N}}{\xi_{\rm NT}}\cdot\frac{\sin(d_{\rm N}/2\xi_{\rm N})}{\sin(d_{\rm NT}/2\xi_{\rm NT})}\ . \tag{5c}$$

Then, $b$ is determined so that the free energy density is minimized. This free energy density is reduced to zero at the upper critical field. The upper critical field or the corresponding flux line spacing is determined from this condition. Especially when $d_{\rm N}$ and $d_{\rm NT}$ are sufficiently smaller than $\xi_{\rm N}$ and $\xi_{\rm NT}$, respectively, the upper critical field is reduced to

$$B_{\rm c2} = B_{\rm c2}^{\rm NT}\cdot\frac{1 + d_{\rm N}/d_{\rm NT}}{1 + d_{\rm N}\xi_{\rm N}^2/d_{\rm NT}\xi_{\rm NT}^2}, \tag{6}$$

where $B_{\rm c2}^{\rm NT}$ is the upper critical field of Nb-Ti. Figure 2 shows the dependence of the upper critical field on thickness of layers at $T = 4.2$ K for $d_{\rm N}/d_{\rm NT} = 0.25$, where we assumed as $\xi_{\rm NT} = 5.33$ nm ($B_{\rm c2}^{\rm NT} = 11.6$ T) and $\xi_{\rm N} = 17.3$ nm ($B_{\rm c2}^{\rm N} = 1.10$ T).

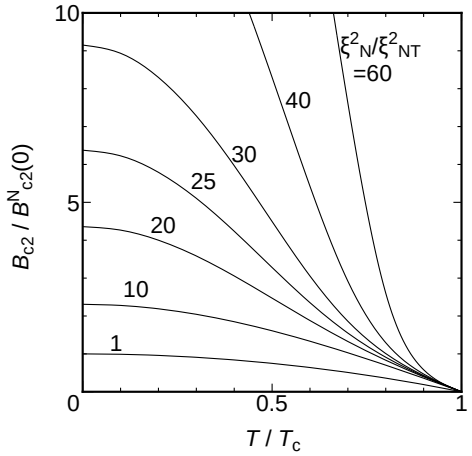


Fig. 3. Temperature dependence of normalized upper critical field for various values of $\xi_{\rm N}^2/\xi_{\rm NT}^2$ for $d_{\rm N} = d_{\rm NT} = 0.75\xi_{\rm N}(0)$: (a) Takahashi-Tachiki theory [3] and (b) present theory.

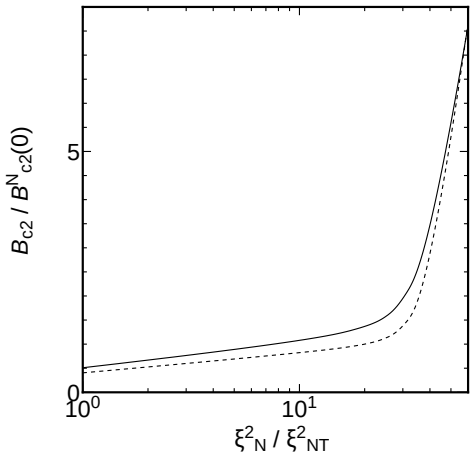


Fig. 4. Dependence of normalized upper critical field on $\xi_{\rm N}^2/\xi_{\rm NT}^2$ for $d_{\rm N} = d_{\rm NT} = 0.75\xi_{\rm N}(0)$. The solid and broken lines are the results of present theory and Takahashi-Tachiki theory [3].

It is seen that the upper critical field approaches asymptotically the value given by Eq. (6) when the thickness is reduced to zero. On the other hand, it is expected to be saturated to the bulk value when the thickness becomes large. However, the approximation in Eq. (3) is not suitable and some modification is necessary.

The upper critical field of multilayer in the normal direction was also calculated by Takahashi and Tachiki [3]. Their result and our corresponding result for the case of $d_{\rm N} = d_{\rm NT} = 0.75\xi_{\rm N}(0)$ are shown in Fig. 3(a) and (b), respectively, where $\xi_{\rm N}^2/\xi_{\rm NT}^2$ corresponds to the ratio of the diffusion constants, $D_{\rm N}/D_{\rm S}$, in Ref. [3]. It is seen that the agreement is satisfactory except for the low temperature region. In Fig. 4 the two results are compared with respect to the dependence on $\xi_{\rm N}^2/\xi_{\rm NT}^2$ at the reduced temperature of $T/T_{\rm c} = 0.7$. Thus, the obtained good agreement shows that the present straightforward method using a proper approximate forluma is useful for describing the superconducting property in the multilayer. It can be expected that this method is also useful for a detailed calculation on more complicated characteristics such as the pinning energy.

## CONCLUSION

The upper critical field of multilayer of superconducting Nb-Ti and Nb was calculated using the Ginzburg-Landau theory with a simple formula of spatial variation in the order parameter. The obtained result agrees well with the microscopic calculation by Takahashi and Tachiki. It is concluded that the present simple method describes correctly the superconducting property of the multilayer.

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